Extended Abstract

A Priori and Adaptive Reliable Routing in Stochastic Dynamic Networks with Correlations

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INTRODUCTION

Transportation networks and their performance are susceptible to fluctuations (Mahmassani et al., 2014) leading to uncertainty in network travel times. Thus, future travel times in the network can be known a priory only with uncertainty, where systematic time-dependence of travel times levels leads random variables travel times with time-varying distributions. Such stochastic, time-varying (STV) networks (Miller-Hooks and Mahmassani, 2000) can provide a more realistic and detailed representation of the network conditions, allowing for critical routing decision to incorporate knowledge of the network uncertainties. Furthermore, the network structure and presence of travel patterns impose spatial dependencies or correlations between link travel times (Kim and Mahmassani, 2015).

Reliability-based routing has been studied extensively, initially on stochastic stationary networks and then STV networks. The seminal work by Frank (1969) presents a closed form solution for travel time probability distributions on shortest paths in stochastic stationary networks. In STV networks, studies by Miller-Hooks and Mahmassani focus on finding least expected time (LET) paths (2000), least possible travel time paths (1998a), and consider different definitions of optimality with proposed label correcting algorithms (Miller-Hooks, 1999; Miller-Hooks and Mahmassani, 2003, 1998b). A number of studies have considered correlations between link travel times (Chen et al., 2012; Fan et al., 2005; Huang and Gao, 2012; Nie and Wu, 2009; Zockaie et al., 2016, 2015, 2013) representing spatial correlations with known transition probabilities or joint link travel time distributions.

Furthermore, path finding solutions in stochastic networks are inherently uncertain, and in networks with spatio-temporal dependencies knowledge of the future states of the network may improve and lead to new solutions for the optimal path en-route (Gao and Chabini, 2006; Huang and Gao, 2018; Miller-Hooks and Mahmassani, 2003; Pretolani et al., 2009). The task of adaptive routing solutions in stochastic networks is to find a routing strategy (or policy) as a set of paths that can be taken based on decisions made at each intermediate node en-route.

The problem considered in this study is one of finding time-dependent reliable least-time paths, for a specified reliability-based objective, both for the a priori path finding and adaptive routing context.

METHODOLOGY

The solution methodology is a generalized 2-stage approach for finding exact and approximate solutions for both a priori and adaptive decisions in STV networks spatio-temporal dependencies. Given a set of departure times and a specified origin node, Stage 1 generates the eligible paths with a specified risk-tolerance parameter, and Stage 2 uses the reduced network and builds the optimal trajectory-adaptive routing strategy with its travel time distribution. The path travel time distributions are estimated using a Monte-Carlo simulation-based approach assuming the ability to perform conditional sampling for the correlated link travel times.

Stage 1: Generating Eligible Paths

The framework for generating a priori eligible paths utilizes a multiple labeling approach, adapted from early work on the multicriteria shortest path problem (Martins, 1984) and a framework for a priori Pareto-optimal paths assuming independent link travel times in STV networks (Miller-Hooks and Mahmassani, 2003, 1998c).

In this work, we establish a stochastic criterion for path eligibility that tests the upper bound of the probability for a given a priori path to contribute to the optimal strategy. This criterion replaces the use of Bellman's principle in a path search algorithm, which is invalidated both for optimality and stochastic dominance in the context of STV networks with correlations. The path eligibility criterion uses an adjustable risk-tolerance parameter ϵ so that as ϵ increases so does the number of eliminated a priori paths, yielding a more constrained and thus sub-optimal strategy. Setting this parameter to 0 will yield the optimal solution, while still eliminating potentially excessive searching of the network by eliminating sub-paths that have no likelihood of resulting in optimal paths.

Stage 2: Finding Optimal Routing Strategies

Once the eligible paths have been generated from stage one, an a priori solution can be found directly. For adaptive routing strategies, the second stage consists of using the eligible paths to build the optimal trajectory-adaptive strategy for one or more optimality criteria.

In this algorithm, the set of eligible paths is read from each destination to the origin node. At each branching node, conditional distributions for the sub-strategy at that node are obtained, the objective value(s) are computed and the appropriate extension path for each sampled realization of the network is selected. Given that the reading of the network and the simulation are performed once, solving the optimal routing strategy for multiple objectives does not have a significant impact on the computational time and effort. However, since different objectives can be expected to result in different strategy distributions, strategy building with multiple criteria generates and holds a larger amount of data.

RESULTS

Numerical experiments were performed on the network of Chicago, with time-dependent and correlated link travel times. Multivariate log-normal distributions were fitted to simulated data in order to allow for the ability of conditional sampling. The numerical experiments were performed for a randomly selected origin node and 34 destinations, a selected set of 5 departure times from 7 AM to 7:20 AM, and 8 values for the risk tolerance parameter, varied between 0 and 0.2, for a total of 1360 numerical tests.

The results from Stage 1, presented in Table 1, show that increasing the value of ϵ results in a decreased number of paths considered eligible across the departure times and destination nodes. For all destination nodes and each departure time, the set of eligible paths selected in Stage 1 always contained the optimal a priori path according to all three of the objective functions.

Table 1. Stage 1 Results: Total and mean numbers of a priori paths

ϵ value	0	0.001	0.005	0.01	0.05	0.1	0.15	0.2
Total number of paths	7310	6375	5761	5352	3581	2287	1563	1134
Mean number of paths for each departure time	1462	1275	1152.2	1070.4	716.2	457.4	312.6	226.8
Mean number of paths for each destination	215.00	187.50	169.44	157.41	105.32	67.26	45.97	33.35

Table 2. Stage 2 Results: Average values for the computational run time, number of evaluation points and optimal objective function value in travel time minutes, across departure times and destination nodes, for the adaptive strategy and a-priori path solutions

	Adaptive Strategy Solution								A priori Solution
ϵ value	0	0.001	0.005	0.01	0.05	0.1	0.15	0.2	–
Run time (s)	739.55	733.59	709.16	690.22	493.65	325.29	219.16	178.70	–
Number of branching nodes (evaluation points)	183.88	160.36	143.91	131.95	81.92	53.25	34.48	25.01	–
Minimum expected travel time [min]	8.42	8.50	8.57	8.57	8.58	8.66	9.02	9.19	10.14
Minimum 80% confidence travel time [min]	12.25	12.37	12.38	12.39	12.60	12.79	13.35	13.85	14.41
Minimum 90% confidence travel time [min]	16.54	16.70	16.83	16.84	17.03	17.15	17.78	17.76	17.83

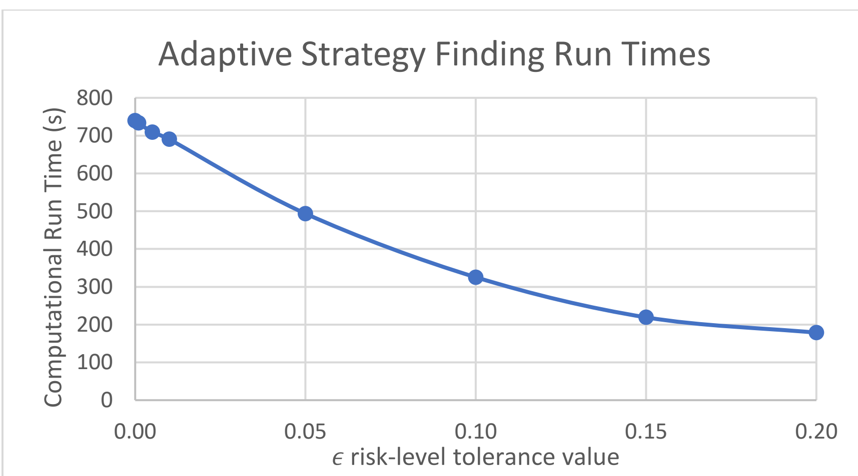

Figure 1. The (average) computational run times for Stage 2 with the different values of ϵ

It can be observed that the computational run time for finding the optimal routing strategy decreases at a near exponential rate with the increase of ϵ. On the other hand, the objective function values for these minimization objectives increases at a near-logarithmic rate with the value of ϵ.

CONCLUSION

This paper presents a 2-stage solution approach the problem of determining optimal a priori paths and adaptive routing strategies in STV networks with generalized spatio-temporal correlations between link travel times.

Numerical tests showed that the average running time of the algorithm reduces near-exponentially with the increase of the risk-tolerance parameter ϵ, while incurring some loss to the value of the objective function relative to the exact solution. The results demonstrate that the parameter ϵ may need to be calibrated for the specific network, data and objectives for computational run times and quality of the routing strategy solution.

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